A Treatment of Magnetized Ferrites Using the FDTD Method

J. A. Pereda, L. A. Vielva, A. Vegas, and A. Prieto

Abstract—**The FDTD method is extended to include magnetized ferrites. The treatment of the ferrite material is based on the equation of motion using Gilbert's approximation of the damping term. The validity of the formulation is verified by applying it to the calculation of propagation constants in waveguides containing ferrites with transverse magnetization. The results for a rectangular waveguide filled with ferrites and those for a rectangular waveguide loaded with a centered ferrite slab are compared with the exact ones.**

I. Introduction

The FINITE-DIFFERENCE time-domain (FDTD) method is a powerful and flexible numerical method capable of solving not only radiation and scattering problems but also ones involving guiding and eigenvalues. The FDTD method was initially proposed to handle isotropic and nondispersive materials [1]. Later extensions have made it possible to apply the method to anisotropic materials, which are characterized by diagonal tensors [2], and also to dispersive materials [3]–[5]. This letter presents a new extension of the FDTD method to include magnetized ferrites. In addition to their highly dispersive nature, these materials are characterized by tensorial permeability with nonzero off-diagonal elements. Ferrites are basic materials in the development of nonreciprocal devices such as circulators and phase shifters. The analysis of structures with magnetized ferrites is normally very complex and in most cases does not admit an analytical solution. Consequently, the development of new numerical techniques which are capable of analyzing these structures is of great interest.

II. Formulation

As for isotropic media, the formulation is based on the direct discretization of Maxwell's curl equations in time domain,

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} x \vec{E} \tag{1}$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon}\vec{\nabla} x \vec{H}, \tag{2}$$

where $\vec{E}$ is the electric field, $\vec{H}$ the magnetic field, $\vec{B}$ the magnetic induction and ϵ the permittivity of the medium.

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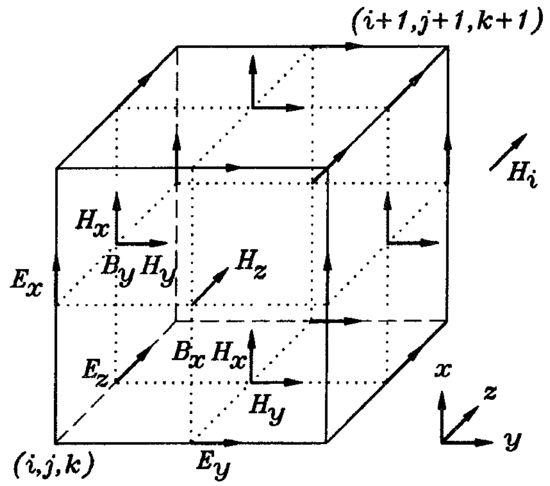

Fig. 1. Three-dimensional extended Yee mesh for the analysis of magnetized ferrites with dc magnetic field applied in the z direction.

To include the ferrite material, it is assumed that the ferrite is saturated by the application of a dc magnetic field in the z direction, $\vec{H}_i = H_i \vec{a}_z$. The interaction of the total magnetic field with the ferrite is described, from a macroscopic point of view, by the equation of motion with Gilbert's approximation of the damping term [6]. Furthermore, assuming a small signal aproximation, i.e. assuming $H_i \gg H$ and $M_s \gg M$, and taking into account the constitutive relationship for ferrites, the equation of motion is expressed, in scalar form, as

$$\frac{\partial B_x}{\partial t} - \mu_0 \frac{\partial H_x}{\partial t} = -\gamma\mu_0 (H_i B_y - (M_s + H_i)\mu_0 H_y) - \alpha\left(\frac{\partial B_y}{\partial t} - \mu_0 \frac{\partial H_v}{\partial t}\right), \tag{3a}$$

$$\frac{\partial B_y}{\partial t} - \mu_0 \frac{\partial H_y}{\partial t} = -\gamma\mu_0 ((M_s + H_i)\mu_0 H_x - H_i B_x) + \alpha\left(\frac{\partial B_y}{\partial t} - \mu_0 \frac{\partial H_x}{\partial t}\right), \tag{3b}$$

where γ is the gyromagnetic ratio, α the damping constant, M_s the saturation magnetization and μ_0 the permeability of a vacuum.

The components H_x and H_y of the magnetic field are coupled due to application of a dc magnetic field in the z direction; hence, these two components must be discretized at the same points of the space and at the same instant of time. Furthermore, the H field and the B field must be discretized at the same instant of time. Taking these considerations into account, Yee's unit mesh [1] is modified as shown in Fig. 1.

Equations (1) and (2) are discretized as in the isotropic case [1]. Equations (3a) and (3b) are discretized by using central finite differences and linear interpolation. Thus, by first discretizing and then decoupling (3a) and (3b), we obtain

$$\begin{aligned} H_x^{n+(1/2)} =& f_0 H_x^{n-(1/2)} + f_1 B_x^{n+(1/2)} + f_2 B_x^{n-(1/2)} \\ &+ f_3 B_y^{n+(1/2)} + f_4 B_y^{n-(1/2)} + f_5 H_y^{n-(1/2)}, \end{aligned} \tag{4a}$$

$$\begin{aligned} H_y^{n+(1/2)} =& f_0 H_y^{n-(1/2)} + f_1 B_y^{n+(1/2)} + f_2 B_y^{n-(1/2)} \\ &- f_3 B_x^{n+(1/2)} - f_4 B_x^{n-(1/2)} - f_5 H_x^{n-(1/2)}, \end{aligned} \tag{4b}$$

where the constants $f_i (i = 0, \cdots, 5)$ are functions of M_s, α, H_i, and the time step Δt.

The new FDTD algorithm for magnetized ferrites has the following steps in each time iteration.

1) $B_x^{n+(1/2)}, B_y^{n+(1/2)}$, and $H_z^{n+(1/2)}$ are calculated by using the discretized version of (1).
2) $H_x^{n+(1/2)}$ and $H_y^{n+(1/2)}$ are calculated by using (4), where $H_x^{n-(1/2)}$, $H_y^{n-(1/2)}$, $B_x^{n-(1/2)}$ and $B_y^{n-(1/2)}$ are obtained from the previous iteration, and $B_x^{n+(1/2)}$ and $B_y^{n+(1/2)}$ are obtained from step 1). As can be seen in Fig. 1, equations (4) are discretized at mesh points where both H_x and H_y are available, but only one component of the B field (B_y or B_x) is known. The other component is calculated by using linear interpolation. For example, at the point $(i, j+\frac{1}{2}, k+\frac{1}{2})$ (see Fig. 1), B_y is calculated as the equation located at the bottom of the page.
3) E_x^{n+1}, E_y^{n+1}, and E_z^{n+1} are calculated by using the discretized version of (2).

III. Validation

The new algorithm has been applied to the determination of propagation constants in waveguides containing ferrites with transverse magnetization. In general, this is a three-dimensional problem that can be reduced to an equivalent two-dimensional problem assuming that the fields have the form $F(x, y, z, t) = F(x, z, t) \exp(-j\beta y)$ [7], where y is the direction of propagation and β the phase constant of the considered mode. When applying the FDTD method to the calculation of dispersion characteristics, the phase constant β is fixed as an input parameter. The resonant frequencies and quality factors of the resonant modes of the waveguide's transverse section are calculated by applying Prony's method to the time domain response [8]. Each pair of resonant frequencies and quality factors (f_i, Q_i) corresponds to one excited propagating mode, which has the previously fixed value of β at the frequency f_i, and an attenuation constant of

$$\alpha_i' = \frac{\pi f_i}{Q_i v_{gi}},$$

where v_{gi} is the group velocity of the mode.

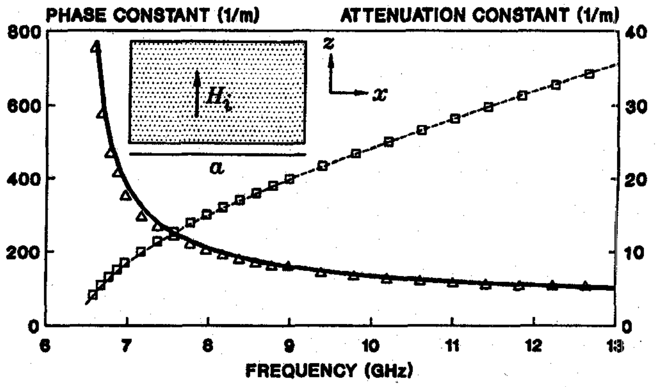

Fig. 2. Phase and attenuation constants of the TE_{10} mode of a ferrite-filled rectangular waveguide. $a = 22.86$ mm, $\epsilon_r = 9$, $4\pi M_s = 2000$ $G.$, $\alpha = 0.02$ and $H_i = 200$ Oe. Phase constant) Dashed line: exact; Squares: FDTD ($\Delta x = a/10$). Attenuation constant) Solid line: exact; Triangles: FDTD ($\Delta x = a/10$).

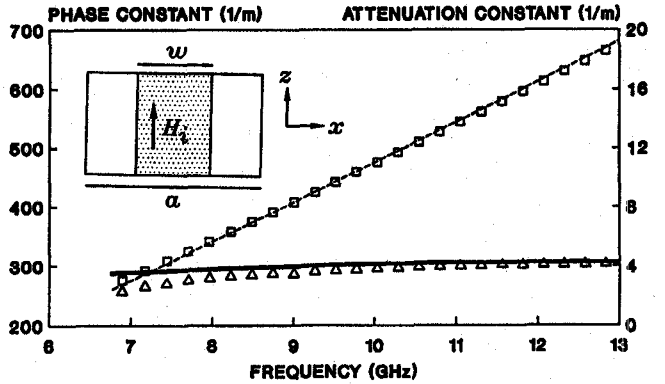

Fig. 3. Phase and attenuation constants of the TE_{10} mode of a rectangular waveguide loaded with a centered ferrite slab. $a = 22.86$ mm, $a/w = 3$, $\epsilon_r = 9$, $4\pi M_s = 2000$ $G.$, $\alpha = 0.02$ and $H_i = 200$ Oe. Phase constant) Dashed line: exact; Squares: FDTD ($\Delta x = a/12$). Attenuation constant) Solid line: exact; Triangles: FDTD ($\Delta x = a/12$).

The results obtained for the propagation constants of the TE_{10} mode are compared with the exact ones [6] for both cases: a rectangular waveguide filled with ferrites and a rectangular waveguide loaded with a centered ferrite slab. The results are shown in Figs. 2 and 3, respectively. It can be seen that the agreement is good.

IV. Conclusion

The finite-difference time-domain method has been extended to include magnetized ferrites. The resulting new algorithm has the same advantages as the FDTD method for isotropic materials: It is flexible, conceptually simple, and easy

$$B_y(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}) = \frac{B_y(i+\frac{1}{2}, j, k+\frac{1}{2}) + B_y(i+\frac{1}{2}, j+1, k+\frac{1}{2}) + B_y(i-\frac{1}{2}, j, k+\frac{1}{2}) + B_y(i-\frac{1}{2}, j+1, k+\frac{1}{2})}{4}$$

to implement. Furthermore, since it is a time-domain method, results for a large frequency bandwidth can be obtained from a single computer simulation. To show the validity of the new algorithm, it has been applied to the calculation of phase and attenuation constants in waveguides containing ferrites with transverse magnetization. The results have been compared with the exact ones and there is good agreement. Apart from the errors involved in FDTD and Prony's methods, it is important to notice that in the calculation of attenuation constants there is an additional source of error which is a consequence of calculating the group velocity from the $\beta - f$ curve.

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